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ON A CASE OF THE APPLICATION OF THE LIMIT OF THE D-SUBDIVISION TO ESTIMATE THE QUALITY OF A SYSTEM OF AUTOMATIC REGULATION

By M. V. Meierov

(Translated by M. Friedman from Izvestia #12, 1950 pp. 1789–1794)

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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In the work of V. V. Solodovnikov (1,2) is reproduced a series of properties of the real frequency characteristics of a closed loop system by means of which it is possible to produce certain estimates of the ~~length~~ (duration) of the transform process and of the amount of regulation.*

The difficulty arising from the use of the frequency method is connected with the necessity of constructing real frequency characteristics of a closed loop. These difficulties are increased considerably if it is necessary to explain the influence of any parameter on the character of the transfer regime since ~~here~~ (for this) a whole family of real frequency characteristics should be built.

In the present work: (1) an effective method is given for the construction of the real and other frequency characteristics of a closed loop by means of the boundary of the D-subdivision (3), (the curve separating the stability region on the plane of the complex amplification coefficient); since the boundary of the D-subdivision is constructed to study stability then the practical construction of real frequency characteristics is not connected with the outlay of time; (2) the possibility is shown of producing

*Note that these estimates are sufficient (not necessary and sufficient) therefore, in practical application of the method, failure of certain properties of the frequency characteristics still does not denote unsuitable systems.

a preliminary estimate of the character of the transform process by means of conditions (1,2) immediately on the boundary of the D-subdivision without having recourse to the construction of real frequency characteristics; (3) the establishment of how the magnitude of the general coefficients of the system is connected to the conditions (1,2) and a method is given of choosing the magnitude of the general amplification coefficient without requiring the construction of a family of curves.

1. Construction of Frequency Characteristics of a Closed Loop From the Boundaries of the D-Subdivision.*

In the sequel, we will consider the case when in a system of automatic regulation a disturbance in the form of a unit impulse with zero initial conditions is supplied. In this case the amplitude-phase characteristics of the closed loop may be expressed in the following way:

$$K(j\omega) = \frac{Kw(j\omega)}{1+Kw(j\omega)} \tag{1.1}$$

where $w(j\omega) = \dfrac{S(j\omega)}{Q(j\omega)}$ is the equation of the amplitude phase characteristic of an open loop.

Let us represent formula (1.1) in another form.

Let us divide numerator and denominator of the right side of (1.1) by $w(j\omega)$; we obtain

$$K(j\omega) = \frac{K}{\dfrac{1}{w(j\omega)} + K} = \frac{K}{\dfrac{Q(j\omega)}{S(j\omega)} + K} \tag{1.2}$$

*The D-subdivision separates the region, in the plane of the complex parameter, with an identical number of roots to the left of the imaginary axis. Here and in the sequel, note that only that part of the boundary of the D-subdivision is used which separates the stability region.)

It is not difficult to show that $\frac{Q(j\omega)}{S(j\omega)}$ with a minus sign represents the equation of the boundary of the D-subdivision (3) with respect to the parameter.* Therefore, the characteristic equation may be found from the relation

$$1 + Kw(p) = 0 \tag{1.3}$$

Substituting for $w(p)$ its value, we obtain

$$1 + K\frac{S(p)}{Q(p)} = 0$$

or

$$Q(p) + KS(p) = 0 \tag{1.4}$$

from which the equation of the boundary of the D-subdivision with respect to K is

$$K = -\frac{Q(j\omega)}{S(j\omega)} \tag{1.5}$$

Let us explain the geometric sense of equation (1.2). In figure 1 is shown the curve of the boundary of the D-subdivision with respect to the parameter K. The letters $a l$ separate the region of the K values in which the system is stable.

The segment $\overline{ab} = K$. The segment $\overline{aB} = -\frac{Q(j\omega_1)}{S(j\omega_1)}$ for the corresponding frequency ω_1; and the segment $\overline{bB}$ is the sum of $K + \frac{Q(j\omega)}{S(j\omega)}$. Hence it is clear that the quotient ~~part~~ of the division of the segment $\overline{aB}$ by ~~on~~ the segment $\overline{bB}$ gives the amplitude ~~value~~ of (1.2) for a given frequency ω_1.

*In the work (4) an inverse frequency characteristic is applied to determine the magnitude of the amplification coefficient, for which there occurs a given peak on the amplitude frequency characteristic. The D-subdivision with respect to the general amplification coefficient differs from the inverse frequency characteristic only by a scale factor.

Phase of (1.2) will be determined by the argument of the denominator and for a given frequency equals the angle of rotation of the vector $\overline{bB}$ counter-clockwise, i.e. the angle $\alpha(\omega_1)$ in figure 1. In the same way, the phase of (1.2) for any frequency ω_1 is found immediately from the drawing.

Having the amplitude and phase it is possible, for any frequency, to calculate, by means of the known relations, the real and imaginary frequency characteristic of the closed loop. The specified frequency characteristics may be found considerably simpler immediately from the drawing. Actually, the real frequency characteristic is found from the equation

$$R(\omega) = H(\omega)\cos\alpha(\omega) = \frac{K}{\left|K+\frac{Q(j\omega)}{S(j\omega)}\right|}\cos\alpha(\omega) = \frac{\overline{ab}}{\overline{bB}}\cos\alpha(\omega) \qquad (1.6)$$

where $H(\omega)$ is the amplitude of $K(j\omega)$.

Let us drop a perpendicular from the point a on the vector $\overline{bB}$; then the segment

$$\overline{bg} = \overline{ab}\cos\alpha(\omega_1) \qquad (1.7)$$

Therefore, for given frequency the ratio of the segments $\overline{bg}|\overline{bB}$ gives the value of the real frequency characteristic. To find the value of the real frequency characteristic for any frequency ω_n (fig. 1) it is necessary to drop a perpendicular from the point a on the line connecting the point b and ω_n (in fig. 1, the segment $\overline{bc}$); then the real frequency characteristic for this frequency is determined from the ratio

$$\frac{\overline{bN}}{\overline{bc}} \qquad (1.8)$$

The imaginary frequency characteristic for the frequencies ω_1 and ω_n are determined respectively from the division of the segments

$$\frac{\overline{ag}}{\overline{bB}} \text{ and } \frac{\overline{aN}}{\overline{bc}} \tag{1.9}$$

because

$$P(\omega) = \frac{\overline{ab}}{\overline{bB}} \sin \alpha(\omega_1) \tag{1.10}$$

In the same way, there may always immediately be constructed simply all the frequency characteristics of a closed system of regulation.

2. <u>Certain Estimates of the Quality of a Transfer Process of a System by Means of the Boundary of the D-Subdivision</u>.

Let us reproduce the well-known [1,2] properties of the real frequency characteristic and find the equivalent properties with respect to the boundary of the D-subdivision. Simultaneously, let us explain how the general coefficient of amplification of the loop influences the specified real frequency characteristics.

The following properties of the real frequency characteristic $R(\omega)$ of a closed loop system of automatic regulation are well known [1,2].

a. In order for the amount of regulation

$$\sigma \text{ percent} = \frac{x_{max} - x(\infty)}{x(\infty)} \times 100 \tag{2.1}$$

not to exceed 18 percent, it is sufficient (but not necessary) that the function $R(\omega)$ be a positive non-increasing continuous function, i.e. it should satisfy the conditions

$$R(\omega) > 0; \qquad \frac{dR(\omega)}{d\omega} \leq 0 \tag{2.2}$$

b. In order that the transfer function $x(t)$ tend monotonically to the steady value $x(\infty)$ it is necessary, but not sufficient, to satisfy the inequality

$$R(\omega) < R(0) \tag{2.3}$$

for all values of ω.

c. The functions $R(\omega)$ having an interval of positiveness ω_c, i.e. the first intersection of the abscissa axis is with the frequency ω_c, correspond to transfer functions with time* of transfer process [1,2]

$$t > \frac{\pi}{\omega_c} \tag{2.4}$$

d. In the case of nonincreasing continuous functions $R(\omega)$, permitting approximation with the aid of trapezoidal frequency characteristics with an interval of passing frequency ω_{oc} and with slope coefficient κ, the time of the transfer process is included between the limits

$$\frac{\pi}{\omega_{oc}} \leq t \leq \frac{4\pi}{\omega_{oc}} \tag{2.5}$$

e. If the function $R(\omega)$ satisfies the condition

$$\left|\frac{R(\omega)}{R(0)}\right| < \frac{0.1}{0.2} \quad \text{for} \quad \omega \geq \omega_c \tag{2.6}$$

Then, as calculations show, to estimate the amount of regulation and the time of the transfer process the form of the function $R(\omega)$ for $\omega > \omega_c$ must not be taken into account.

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*Here and in the sequel the process is considered complete when the regulated quantity differs from its steady value by not more than 5 percent.

In the sequel we will denote the boundary of the D-subdivision by $N(\omega)$. Let us formulate certain properties of $N(\omega)$ equivalent to the properties of the real frequency characteristics represented above.

1. In order that the amount of regulation should not exceed 18 percent, it is sufficient, but not necessary, that $N(\omega)$ satisfy the following conditions (fig. 2): (a) for increasing frequency from 0 to ∞ the absolute value of the vector, produced from the origin to a point on the curve $N(\omega)$ (the vector $a\,\omega_1$) must increase continuously; (b) for a given frequency K_o, the curve $N(\omega)$ must not intersect a circle with center at K_o and radius equal to K_o; (c) the projection of the vector $a\,\omega_1$ on the K axis for $\omega \rightarrow \infty$ must not exceed K_o.

Consequence 1: In order to fulfill the sufficient condition that ~~of~~ regulation not exceed~~ing~~ 18 percent, the amplification coefficient of the loop must not exceed the magnitude ~~amount~~ of the radius K_o of the circle (fig. 2).

Consequence 2: The regulation will not exceed 18 percent for any amplification coefficient if the curve $N(\omega)$ coincides with the axis of the $\bar{K}$ plane and the whole positive real axis of this plane is a region of stability.

2. In order that the transfer function $x(t)$ tend monotonically to the steady value $x(\infty)$ it is necessary that the curve $N(\omega)$ does not intersect the circle with radius K_o and ~~on~~ center at K_o (fig. 3).

Consequence: If by means of the condition of (adjustment) ~~accuracy~~ the amplification coefficient of the loop is larger than the radius K_o, then the transfer process will not be monotonic.

3. The interval of positiveness is defined by the frequency ω_c (fig. 4) where the curve $N(\omega)$ is intersected by a perpendicular produced to the line K..This system has the time of the transfer regime

$$t > \frac{\pi}{\omega_c} \tag{2.7}$$

4. If in the first part, the curve $N(\omega)$ is sufficiently close to coincidence with an arc (not greater than 30°) of the circle produced from the point K_o of radius K_o, and in the later section, the distance between the circle and the curve $N(\omega)$ grows continuously (fig. 5) then the time of the transfer process is included between the limits

$$\frac{\pi}{\omega_{co}} \leq t \leq \frac{4\pi}{\omega_{co}} \tag{2.8}$$

where ω_{co} is the interval of positiveness.

5. If the absolute value of the vector $\overrightarrow{a\,\omega_1}$ increases continuously with increasing frequency (fig. 6) and $\frac{K_o}{K} < \frac{0.1}{0.2}$, then as calculations show, to estimate the amount of regulation and the time of the transfer process the form of the curve $N(\omega)$ for $\omega > \omega_c$ may not be taken into account.

The nature of the particular, formulated properties of the curve $N(\omega)$ is ~~are~~ ~~be~~ such that they are immediately related to the general amplification coefficient of the loop. For these conditions, the latter may be chosen such that they guarantee the fulfillment of these conditions (if, in general, fulfillment of the specified sufficient conditions in a given concrete case is possible).

3. Application of Calculation to a Regulation System.

Let us consider the calculation of a system of automatic regulation.

Let us ~~admit~~ assume that the object of regulation is a constant current motor with an independent exciter of 1.5 KVA power and 120V-voltage. The motor is loaded with a static load the ~~amount~~ magnitude of which is independent of the speed of rotation. By the character of the technological process, the speed of rotation of the motor must be kept constant with an accuracy not less than 0.125 percent; moreover, the system must admit wide limits of variation ~~of~~ from the steady value of the number of turns. Additional technical requirements are (a) in the process steady regulations must not exceed 10-12 percent with the condition that the disturbance be given in the form of a unit pulse; (b) the time of regulation must not exceed 0.6 second. Here let us assume the regulation process completed when the number of turns differs from the prescribed value by not more than 5 percent.

Choice of Basic Elements of Regulation System

In order to guarantee the possibility of varying the steady number of turns ~~in~~ between wide limits, we will accomplish the regulation action on the voltage of the armature of a motor with constant current excitation.

As a voltage source to supply the motor armature we choose an electromachine booster (EMB). This choice is dependent on these characteristics of the EMB which may be used effectively in the considered case (large amplification coefficient with respect to voltage and power, very small power on input, etc.)

As a measuring element, we choose a tacho-generator, the electromotive force (e.m.f.) of which is equal to the constant voltage U_{g1}. Between the measuring element and the EMB is included an electronic amplifier. The necessity of the electron amplifier ~~depends~~ is specified, first, ~~such~~ so that the system may possess large coefficient of amplification by means of the ~~accuracy~~ adjustment condition and, second, that the voltage at the input of the tacho-generator be proportional to the turns only in the case when it works no-load. With the inclusion of the tacho-generator voltage in the cathode circuit of the vacuum tube amplifier it may be considered that the voltage of the tacho-generator equals its e.m.f. and that it is directly ~~strictly~~ proportional to the rotational speed of the motor.

Let us obtain a regulating system consisting of the basic elements (fig. 7) which work in the following way. The difference of the tacho-generator e.m.f. and the calibration voltage $U_{\varepsilon 1}$ is fed to the input of the electron amplifier. The winding of exciter (EMB) is supplied from the difference of the output voltage U_y of the amplifier and the calibration voltage $U_{\varepsilon 1}$. In the steady regime, the difference $U_{\varepsilon 1} - \epsilon_T$ forms the voltage input to the EMB which guarantees the prescribed ~~preassigned~~ value of the number of turns. Let us assume that in the result, the variation of loading on number of turns varies, then the e.m.f. ϵ_T of the tacho-generator varies and correspondingly the difference $U_{\varepsilon 1} - \epsilon_T$ which expresses the necessary variation of the voltage on the EMB and correspondingly guarantees the proper variation of the number of turns of the motor.

Block Diagram of a Regulation System

Let us compose the block diagram of the obtained system of automatic regulation.

The equation of the motor with a regulated number of its turns acting on the voltage of an armature with constant stream of excitation may be described

$$(T_M T_S P^2 + T_M P + 1)\, n = K U_a \tag{3.1}$$

where $\dfrac{GD^2 R_S}{375\, C_M C_e \varphi^2} = T_M$ is the electro-mechanical time constant; $T_S = \dfrac{L_S}{R_S}$ is the time constant of the armature; $K = 1/C_e \varphi$ is the amplification coefficient.

Independently of the value of the time constants T_M and T_S ~~of~~ the motor it may be ~~is possible~~ ~~to~~ represented by one oscillatory section or two successive connected ~~tions~~ relaxation sections.

In the considered case $T_M = 0.5$ second; $T_S = 0.01$ second; $K = 1$. It is not difficult to see that here the motor ~~may~~ be represented by two successive connected relaxation sections with time constant $T_3 = 0.5$ second, $T_4 = 0.01$ second. Actually, the roots of the left side of the equation (3.1) will be $\alpha_1 = -2$, $\alpha_2 = -98$. Therefore $T_3 = -1/\alpha_1 = 1/2 = 0.5$ second, and $T_4 = 1/\alpha_2 = 1/98 \approx 0.01$ second.

Let us consider the electro-machine amplifier completely compensated and consequently it is possible to represent it by two successive connected relaxation sections with time constants T_1 and T_2 and amplification coefficients K_1 and K_2.

In the considered example the time constant of the short-circuited loop is $T_2 = 0.1$ second, and the time constant of the loop of the EMB exciter is $T_1 = 0.1$ second.* The general amplification coefficient of the EMB is $K_1K_2=10$.

In figure 8 is represented the block diagram of a system consisting only of the basic elements. There is obtained a single-loop system consisting of four successive relaxation sections.

The characteristic equation which is obtained as the result of multiplying the operators of the separate sections is written:

$$(1+T_1P)\,(1+T_2P)\,(1+T_3P)\,(1+T_4P) + K_1\,K_2\,K_3\,K\,K_y = 0 \qquad (3.2)$$

*The time constant, besides the inductiveness of its excitation winding, takes into account the inertia of the electron amplifier.

Here K_s is the gear ratio of the reducer, joining the shafts of the motor and tacho-generator; K_y is the coefficient of amplification of the amplifier.

With respect to the condition of ~~accuracy~~ adjustment the general amplification coefficient is

$$K = K_1 K_2 K_3 K_4 K K_y = \frac{1}{0.00125} = 800$$

Substituting instead of the time constants their values and considering the general amplification coefficient equal to 800, we obtain:

$$0.00005\, P^4 + 0.0061\, P^3 + 0.117\, P^2 + 0.71\, P + 800 = 0 \qquad (3.3)$$

It is not difficult to show that the system is unstable for this amplification coefficient.

4. System With An Introduced Stabilizing Section

The system obtained in the preceding paragraph doesn't satisfy the stability condition, consequently it is necessary to stabilize it. To do this we include a stabilizing section in the system.

The most practicable ~~point of view~~ method of accomplishing this technically is the stabilizing section with operator equation

$$\frac{T_5 P}{1+T_5 P} \qquad (4.1)$$

As is well known [5,6], with the aid of this stabilizing section, it is possible to obtain a system which in principle remains stable for an amplification coefficient as large as one pleases if two relaxation sections are connected around it, with sufficiently large coefficients of amplification.

As stabilization sections we choose networks of resistance R and capacity C and connect them as shown in figure 9. To the input of the stabilizing section the voltage of the EMB is supplied and the output voltage of the stabilizing section is fed to the input of the electron amplifier.

The block diagram of the full regulation system is represented in figure 10. The stabilization section is connected around the first two relaxation members connecting the electron amplifier. This scheme with the correct choice of the parameters may remain stable for amplification coefficient as high as one pleases.

The characteristic equation of the system is

$$(1+T_1P)\ (1+T_2P)\ (1+T_3P)\ (1+T_4P)\ (1+T_5P) + K_1K_2K_y\ [T_5P(1+T_3P)(1+T_4P) + (1+T_5P)\ K\ K_3] = 0 \qquad (4.2)$$

Here the degenerate equation [5,6] will be

$$T_5P\ (1+T_3P)\ (1+T_4P) + K\ K_3\ (1+T_5P) = 0 \qquad (4.3)$$

and it must satisfy the Routh-Hurwitz condition [5,6].

In equation (4.3) T_3 and T_4 are given quantities. As regards the time constant T_5 of the stabilizing section and the amplification coefficient K_3 (of the gear ratio of the reducer of the motor shaft to the tacho-generator shaft), it is necessary to choose these quantities. Here consider not only the necessity of obtaining ~~not only~~ stability but the quality of the system.

Let us choose the time constant of the stabilization section such that equation (4.3) satisfies the stability condition for any value of the amplification coefficient $K K_3$.

This will occur if

$$T_5 > \frac{T_3 T_4}{T_3 + T_4} \tag{4.4}$$

as it is not difficult to confirm immediately by calculations.

In our case

$$\frac{T_3 T_4}{T_3 + T_4} = \frac{(0.01)(0.5)}{0.5 + 0.01} = \frac{0.005}{0.51} = 0.01$$

Let us choose $T_5 = 0.2$ second.

There remains for us to choose the amplification coefficients $K_3 K$ and $K_1 K_2 K_y$. Let us select the amplification with respect to the voltage of the electron amplifier: $K_y = 20$, then $K_1 K_2 K_y = 10.20 = 200$.

Constructing the boundary of the D-subdivision with respect to the general amplification coefficient and separating the region of stability, it is necessary to choose, in this region, that value of K which, on the one hand, satisfies all conditions of static adjustment and, on the other hand, satisfies the qualitative requirements, as formulated in the technical specification.

5. <u>Choice of General Amplification Coefficient and Certain Estimates of the Character of the Transfer Process</u>.

Let us construct a boundary of the D-subdivision with respect to the general amplification coefficient. The amplification coefficients of the EMB and the electron amplifier were chosen as $K_1 K_2 K_y = 200$. The general amplification coefficient $K_o = K_1 K_2 K_y K_3 K = 200 K_3 K$.

From formula (4.2) the equation of the boundary of the D-subdivision with respect to the general amplification coefficient may be described as

$$K_o = \frac{(1+0.1\ j\omega)^2(1+0.01\ j\omega)(1+0.5\ j\omega)(1+0.2\ j\omega)+200\times(0.2\ j\omega)(1+0.5\ j\omega)(1+0.01\ j\omega)}{1 + 0.2\ j\omega} \tag{5.1}$$

In figure 11 is reproduced the curve of the boundary of the D-subdivision constructed by means of equation (5.1). The region of the value of the general amplification coefficient K_o, for which the system is stable, is denoted by the letters ~~ag~~ al. The largest value of the general amplification coefficient, for which the system is stable, is $K_{cr} = 5250$, considerably larger than the value~~s~~ of the amplification coefficient necessary to guarantee the static adjustment of regulation.

By the technical requirements the general amplification coefficient must not be less than 800, this is, therefore, the lower bound.

From figure 11, it is clear that if the general amplification coefficient does not exceed 1000, then it is possible, on the basis of property 5 of the $N(\omega)$ curve to discard part of the curve corresponding to the frequencies $\omega > \omega_c$. We see, moreover, that the circle with radius $K = 1000$ does not intersect $N(\omega)$ and that the initial section almost coincides with it.

On the basis of ~~the division~~ sections 1 and 3 it is possible to conclude that regulation will not be larger than 18 percent and the time of regulation will lie between the limits $\frac{\pi}{\omega_{co}} < t < \frac{4\pi}{\omega_{co}}$ or in our case 0.125 sec. $\leq t \leq 0.5$ sec.

By the time the process is completed the technical requirement is satisfied. It is known that the amount of regulation will be less than 18 percent. For preciseness and complete mapping we build the transfer process.

In figure 12 is reproduced the curve of the transfer process from which it is evident that the regulation process is ended by 0.45 second, and the regulation equals 6 percent, which completely satisfies the technical requirements.

CONCLUSIONS

1. There is proved the possibility of applying the boundary of the D-subdivision for a preliminary estimate of the amount of regulation and the time of the transfer process.

2. A ~~The devised~~ method derived of estimating the influence of the magnitude ~~amount~~ of the general amplification coefficient on the character of the disposition of the real frequency characteristic of a closed system of automatic regulation.

REFERENCES

Russian 1. Solodovnikov, V. V.: On an approximate method of studying the dynamics of an automatic regulation system. Izv. No. 12, 1945.

Russian 2. Solodovnikov, V. V.: Analysis of the quality of a follower system by means of its amplitude phase characteristics. Izv. No. 4, 1949.

Russian 3. Neimerk, Y. I.: On the determination of the value of the parameters for which an automatic regulation system is stable. Automatika e Telemekhanika No. 3, 1948.

English 4. Kress, G. B.: The inverse Nyquist plane in servo-mechanism's theory. PJRE vol. 35 No. 12, 1947.

R 5. Meierov, M. V.: Systems of automatic regulation, stable for very large coefficients of amplification. Automatika e Telemekhanika, No. 4, 1947.

R 6. Meierov, M. V.: Principle of constructing an automatic regulation system with small steady error. Automatika e Telemekhanika No. 2, 1949.